Hyperbolic Function

1. Hyperbolic function are defined in terms of exponential functions. Ans: True

2.The properties of hyperbolic function is analogous to the trigonometric function. Ans: True

3.Hyperbolic function cannot be used to model catenaries. Ans: False, (can be)

4. The hyperbolic function $\int \coth u \, du$ is equal to $-\ln[\operatorname{sech} u] + C$. Ans: True

5. The hyperbolic function $\int \cosh u \, du$ is equal to $-\ln[\operatorname{csch} u] + C$. Ans: False, $\sinh u + C$

6. $\int \frac{\cosh x}{\cosh^2 x} dx$ Ans: $\arctan(\sinh(x)) + C$

7. $\int_0^{\ln 4} 4e^x \sinh x \, dx$ Ans: $15 - \ln 16$ or 12.227

8. $\int e^{3x} \sinh x \, dx$ Ans: $\frac{e^{4x} - 2e^{2x}}{8} + C$

9. $\int \frac{\cosh x}{6+5\sinh x} dx$ Ans: $\frac{1}{5}\ln(6 + 5\sinh x) + C$

10. $\int 6e^x \sinh x \, dx$ Ans: $\frac{3}{2}e^{2x} - 3x + C$

Solution:

6. $\int \frac{\cosh x}{\cosh^2 x}$

$= \int \frac{1}{\cosh x} dx$

$= \int \cosh x \cdot \frac{1}{\sinh^2 x + 1} dx$

$u = \sinh x \qquad du = \cosh x$

$dx = \frac{1}{\cosh x} du$

$= \int \frac{1}{u^2+1} du$

$= \arctan(\sinh(x)) + C$

7. $\int_0^{\ln 4} 4e^x \sinh x \, dx$

$= (e^{2x} - 2x)|_0^{\ln 4}$

$= \left[e^{2(\ln 4)} - 2(\ln 4)\right] - \left[e^{2(0)} - 2(0)\right]$

$= (e^{\ln 4^2} - \ln 4^2) - (e^0 - 0)$

$= e^{\ln 16} - \ln 16 - 1$

$= 16 - \ln 16 - 1$

$= 15 - 2\ln 4$ or 12.227

8. $\int e^{3x} \sinh x \, dx$

$\int (\frac{e^{2x}}{4} - \frac{1}{4}) \cdot 2e^{2x} dx$

$u = e^{2x} \qquad du = 2e^{2x}$

$dx = \frac{e^{-2x}}{2} dx$

9. $\int \frac{\cosh x}{6+5\sinh x} dx$

$u = 6 + 5\sinh x, \qquad du = 5\cosh x \, dx,$

$\cosh x \, dx = \frac{du}{5}$

$\int \frac{\cosh x}{6+5\sinh x} dx$

$= \frac{1}{4}\int(u-1)du$

$\int(u-1)du$

$= \int u\,du - \int 1du$

$= \frac{u^2}{2} - u$

$= \frac{1}{4}\int(u-1)du$

$= \frac{u^2}{8} - \frac{u}{4}$

$= \frac{e^{4x}}{8} - \frac{e^{2x}}{4} + C$

$= \frac{e^{4x}-2e^{2x}}{8} + C$

$= \int \frac{\frac{du}{5}}{u}$

$= \frac{1}{5}\int \frac{du}{u}$

$= \frac{1}{5}\ln u + C$

$= \frac{1}{5}\ln(6 + 5\sinh x) + C$

10. $\int 6e^x \sinh x\,dx$

$= \int 6e^x \cdot (\frac{e^x - e^{-x}}{2})\,dx$

$= \int 3(e^{2x} - e^0)\,dx$

$= 3\int(e^{2x} - 1)\,dx$

$= 3\left(\frac{1}{2}e^{2x} - x\right) + C$

$= \frac{3}{2}e^{2x} - 3x + C$